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# A 3-parameter critical power model

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*Keywords:* Anaerobic work capacity; Cycle ergometry; Endurance; Exhaustion; Hyperbolic model.

The critical power test is a well-established procedure that provides estimates of two important parameters characterizing work performance; anaerobic work capacity (AWC) and critical power (CP). The concept proscribes a hyperbolic relationship between power output ($P$) and time to exhaustion ($t$), given by $(P - \mathrm{CP})t = \mathrm{AWC}$. Since evidence now exists that the procedure overestimates CP and underestimates AWC, this study was undertaken to investigate the effect of relaxing the requirement that the time asymptote necessarily be at zero. Using data from a previous study, it is shown that in so doing, (1) a time asymptote significantly less than zero is obtained, (2) significantly smaller estimates of CP and larger estimates of AWC are obtained, (3) a third parameter is introduced that theoretically represents maximal instantaneous power, (4) it implies that the maximal power that could be developed at any instant is proportional to the amount of AWC remaining at that instant, which in turn implies that (5) at exhaustion not necessary all of AWC is consumed.

## 1. Introduction

The critical power test (Hill 1993, Monod and Scherrer 1965, Moritani *et al.* 1981) is actually a series of physical trials to exhaustion, each at different but fixed power settings on the cycle ergometer. The test is intended to provide estimates of two important parameters characterizing work performance. Anaerobic work capacity (AWC, joules) is that parameter representing the aggregate total work that can be performed by the body's limited energy resources (phosphagens, glycolysis resulting in net lactate production, and oxygen stores), regardless of the rate at which these resources are used. The critical power (CP, watts) is that power setting representing the upper limit for prolonged work; a power that could in theory be maintained indefinitely. Both AWC and CP have important implications in the study of humans as a source of mechanical power (Wilkie 1960).

In its original model formulation, Monod and Scherrer (1965) postulated a linear relationship between the total work performed, $W_{\mathrm{tot}}$ (J), and the time to exhaustion at constant power, $t$ (s):

$$W_{\mathrm{tot}} = \mathrm{AWC} + \mathrm{CP}t \tag{1}$$

and experimental data confirmed this model as an adequate description of the process. For constant power $P$ (W), equation (1) can be rewritten as

$$W_{\mathrm{tot}} = Pt = \mathrm{AWC} + \mathrm{CP}t$$

i.e.

$$P = \mathrm{AWC}(1/t) + \mathrm{CP} \tag{2a}$$

or

$$t = \mathrm{AWC}/(\mathrm{P} - \mathrm{CP}) \tag{2b}$$

or

$$(P - CP)t = \mathrm{AWC} \tag{2c}$$

Equations 2b and 2c are alternate formulations often referred to as the (simple) hyperbolic model. The parameter CP is the power asymptote and it is noted that the time asymptote is located at $t = 0$.

This general approach has been adopted for running on the treadmill (Hughson *et al.* 1984), and for swimming (Biggerstaff *et al.* 1992, Wakayoshi *et al.* 1992) in order to estimate an anaerobic 'distance' capacity and the critical 'velocity'. It has also been adopted for kayaking (Ginn and MacKinnon 1989) and could presumably be adopted for other forms of dynamic exercise where work rate can be regulated.

## 2. Inadequacies of the 2-parameter critical power model

Although the critical power test has been regarded as providing a reliable measure of the maximal fatigueless rate of work (Gaesser and Wilson 1988, Nesbelsick-Gullett *et al.* 1988) irrespective of which equation formulation is used (Smith and Hill 1992), it has more recently come under criticism. Despite all subjects in some studies being able to complete at least 24 min at their estimated CP (Overend *et al.* 1992, Poole *et al.* 1988), it is a recurrent finding that the experimentally determined CP seems to obviously overestimate the power that can be maintained continuously. Housh *et al.* (1989) found that eleven out of fourteen of their subjects could not maintain exercise on the cycle ergometer at their estimated CP for 1 h. Average endurance was found to be 33·31 min. Jenkins and Quigley (1990) found six out of eight cyclists and McLellan and Cheung (1992) found 13 out of 14 subjects, unable to maintain CP for 30 min without exhaustion. In the former study (Jenkins and Quigley 1990) the subjects' mean attained power over 30 min was 6·4% below their estimated CP. A very similar conclusion has been reached by Pepper *et al.* (1992) for treadmill running at critical velocity. Average time to exhaustion at estimated CP was found to be even lower, at only 16·43 min. Studies in which participants were specifically directed to exercise as long as possible or to stop at 90 min (Hill *et al.* 1991, Scarborough *et al.* 1991), found longer endurance times averaging 43 and 51 min at estimated CP on their first attempt. The second attempt by the subjects in these studies produced noticeable improvements, averaging 51 and 65 min, respectively. In two of the above studies, (Housh *et al.* 1989, Pepper *et al.* 1992), time to exhaustion at power settings above CP were accurately predicted, and the estimates of AWC were not called into question. However, evidence as to the test-retest reliability of AWC is ambiguous (Gaesser and Wilson 1988, Nebelsick-Gullett *et al.* 1988, Smith and Hill 1993).

The fact that oxygen delivery takes time to reach a steady state has led Vandewalle *et al.* (1989) to question the immediate availability of the CP component in equation 1. As a consequence of this lag, the estimate of AWC derived from the CP test can be shown to be negatively biased. Hence the estimate of CP must be positively biased. However, their adjustment to CP or AWC estimated on this basis does not appear to have been tested in terms of endurance times at these adjusted values. Vandewalle *et al.* (1989) also question the assumption that at the point of exhaustion, all of AWC is used up as is also implied in equation 1. These authors however offer no alternative assumption.

Alternatives to the CP model concept have been developed. These include, for example, the three component bioenergetic model of Morton (1990), and an extension of the two-component hyperbolic model due to Peronnet and Thibault (1989). These do overcome some obvious deficiencies of the simple CP model, but at the cost of considerable complexity and significantly more parameters. Morton's equation takes the form

$$P = \frac{k_0 + k_1 e^{-r_1 t} + k_2 e_{-r_2 t}}{k_3 + k_4 e^{-r_1 t} + k_5 e_{-r_2 t}}$$

while Peronnet and Thibault's takes the form

$$P = [S/t(1 - e^{-t/k_2})] + \frac{1}{t}\int_0^t [\mathrm{BMR} + \mathrm{B}(1 - e^{-\tau/k_1})]\mathrm{d}\tau$$

The remainder of this communication offers a plausible parsimonious extension of the CP model by relaxing the time asymptote constraint. The effect of so doing is investigated by application to data from a previous study.

## 3. Model development

Without loss of mathematical generality we can extend equation 2c allowing a non-zero asymptote at $t = k$ by writing

$$(P - \mathrm{CP}) \cdot (t - k) = \mathrm{AWC} \tag{3a}$$

$$\text{or } t = \mathrm{AWC}/(P - \mathrm{CP}) + k \tag{3b}$$

If in fact this extension is vacuous, we will discover that $k$ is not significantly different from zero, in which case equation 3a reverts to equation 2c. This extension can be visualized in figure 1.

## 4. Illustrative example

The above procedures can be illustrated using the data McLellan and Cheung (1992),

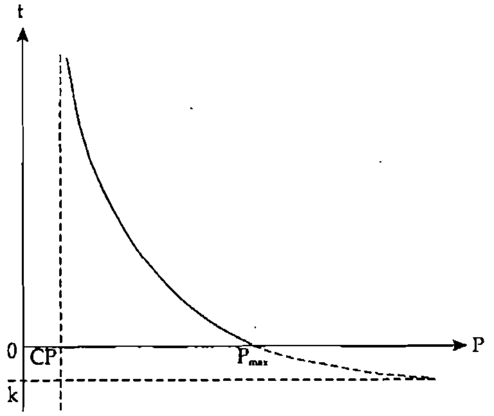

Figure 1. A diagrammatic represention of the 3-parameter CP test model equation. It is represented as a generalized rectangular hyperbola, with verical power asymptote at $P = \mathrm{CP}$, horizontal time asymptote at $t = k = \frac{AWC}{\mathrm{CP}-P_{max}}$, and cutting the power axis at $P = P_{\max}$.

Table 1. Fits of the 2-parameter CP model, giving details of the fits of the standard CP model to the data of McLellan and Cheung (1992).

| Equation (2a) | | | | | | |
|---|---|---|---|---|---|---|
| Subject | AWC (J) | SE$^{\dagger}$ | CP (W) | SE | RSS$^{\dagger}$ | RMS* |
| A | 12579 | 2313 | 254·0 | 14·2 | 9643 | 3214 |
| B | 19127 | 2193 | 317·9 | 8·5 | 21283 | 7094 |
| C | 24492 | 2786 | 277·6 | 9·1 | 5186 | 1723 |
| D | 12075 | 1131 | 328·3 | 6·9 | 57676 | 19225 |
| E | 23430 | 729 | 285·2 | 2·8 | 2249 | 750 |
| F | 14329 | 1338 | 276·0 | 6·7 | 4885 | 1628 |
| G | 16530 | 607 | 275·1 | 2·8 | 1842 | 614 |
| H | 22717 | 1662 | 292·1 | 6·3 | 3434 | 1145 |
| I | 16358 | 528 | 201·8 | 2·3 | 664 | 221 |
| J | 8873 | 1395 | 202·7 | 7·7 | 38002 | 12667 |
| K | 12156 | 682 | 281·6 | 3·9 | 944 | 315 |
| L | 19772 | 435 | 265·5 | 1·8 | 86 | 29 |
| M | 17904 | 835 | 243·0 | 3·2 | 6352 | 2117 |
| N | 15361 | 1344 | 210·7 | 6·8 | 1498 | 499 |

†SE = Standard error of the respective estimates.
†RSS = Residual sum of squares.
*RMS = Residual mean square.

which were used to compare CP with the individual anaerobic threshold. Six data points per subject, one of which was obtained from a randomly chosen repeated test,

Table 2. Fits of the 3-parameter CP model, giving details of the fits of the extended 3-parameter CP model to the data of McLellan and Cheung (1992). Both versions of the model equations 3b and 3c are entertained, producing third parameters $k$ and $P_{max}$.

| Equation [3] | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Subject | AWC | SE$^{\dagger}$ | CP | SE | k (s) | SE | $P_{max}$ (W) | SE | RSS$^{\dagger}$ | RMS* |
| A | 28066 | 6097 | 213·0 | 11·6 | −67·1 | 4·3 | 631 | 85 | 6874 | 3437 |
| B | 72176 | 3684 | 263·6 | 27·9 | −265·9 | 18·9 | 535 | 47 | 3443 | 1721 |
| C | 27472 | 8650 | 271·0 | 24·2 | −10·3 | 1·9 | 2936 | 204 | 3550 | 1775 |
| D | 23750 | 3870 | 308·7 | 16·5 | −73·5 | 5·0 | 632 | 73 | 15387 | 7694 |
| E | 27651 | 8603 | 279·0 | 10·6 | −22·1 | 3·6 | 1528 | 422 | 2055 | 1027 |
| F | 15307 | 5369 | 274·1 | 7·1 | −7·7 | 1·2 | 2252 | 1140 | 2578 | 1289 |
| G | 20360 | 3994 | 268·4 | 5·7 | −24·1 | 1·5 | 1112 | 817 | 475 | 238 |
| H | 27010 | 1188 | 283·9 | 16·5 | −18·8 | 2·2 | 1723 | 577 | 2553 | 1276 |
| I | 18059 | 3839 | 199·1 | 5·6 | −12·7 | 2·7 | 1619 | 388 | 364 | 182 |
| J | 129290 | 23620 | 82·1 | 51·6 | −534·5 | 48·9 | 324 | 25 | 5960 | 2980 |
| K | 12158 | 1011 | 281·4 | 3·8 | ...... | | xxxx | | 934 | 467 |
| L | 20371 | 1982 | 264·3 | 2·8 | −3·2 | 0·9 | 6575 | 2646 | 52 | 26 |
| M | 20928 | 7756 | 238·7 | 9·3 | −22·3 | 1·3 | 1179 | 272 | 3762 | 1881 |
| N | 19170 | 1199 | 202·2 | 21·9 | −22·1 | 1·3 | 1071 | 453 | 708 | 354 |

†SE = Standard error of the respective estimates
†RSS = Residual sum of squares.
*RMS = Residual mean square.
...... = Tending twowards zero.
xxxx = Tending towards infinity.

were used in these estimations. Equation 2a as used by McLellan and Cheung was fitted using SigmaPlot (Jandel Scientific, San Rafael, CA). However, since the coefficient of variation increases with increasing exercise time to exhaustion (McLellan *et al.* 1995), a weighted least squares procedure was utilized. The SigmaPlot manual recommends that in such circumstances the weights $w_i$ be proportional to the square of $y_i$ for the $i$th data point. These were the weights used. The results of these fits are summarized in table 1.

Two versions of the 3-parameter CP model, given by equations 3b and 3c (see below for details) were also fitted to the same data in the same way. The results of these fits are summarized in table 2.

In all but one subject, (K), the 2- and 3-parameter model equations are distinguishable. The time asymptote parameter $k$ is significantly less than zero ($p < 0{\cdot}05$). Estimates of AWC from the 3-parameter model are significantly higher than those from the standard CP test ($p < 0{\cdot}01$). Subjects B and J, whose values from the 3-parameter model are unusually high have been omitted from this comparison. Similarly, the estimates of CP from the 3-parameter model are significantly less ($p < 0{\cdot}05$) than those estimated by McLellan and Cheung (1992). All CP estimates, from either model, are within the normal range for trained and untrained subjects (Hill 1993), and are all significantly greater than zero ($p < 0{\cdot}001$). These minor exceptions are attributed to the observation that although the spread of power settings chosen by McLellan and Cheung (1992) was considered satisfactory for their purpose, they may be less than suitably chosen for the 3-parameter model as discussed below. As a consequence, it will also be noted that the standard errors for both AWC and CP are more often larger with the 3-parameter model, for each subject respectively.

Since the variability of exercise time to exhaustion is not homogeneous (McLellan *et al.* 1995), weighted least squares would be advised a priori. Nevertheless, unweighted least squares was attempted for the 3-parameter model for the purpose

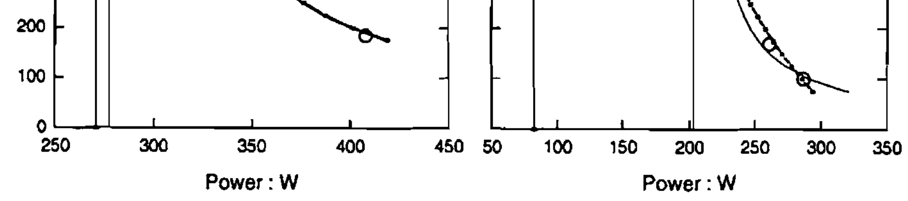

Figure 2. ($t$, $P$) Diagrams for subjects C and J, showing the raw data (open circles), fitted 2-parameter model (thin line), and 3-parameter model (filled squares, joined) for subjects C (left panel) and J (right panel). The two differing estimates of CP for each subject are shown as corresponding lines and vertical symbols.

of comparison. In only eight of the 14 cases could the model be successfully fitted. It therefore appears that successful parameter estimation is sensitive to correct choice of weighting, at least for the data of McLellan and Cheung (1992). Such weighting is therefore recommended.

A comparison between the models can be made, based on the assumption that both models could be used to predict endurance for given power settings. In absolute terms, the residual sum of squares between predicted and observed endurance is significantly smaller ($p < 0.05$) for the 3-parameter model. Even when adjusted for the extra parameter, the residual mean square remains significantly smaller ($p < 0{\cdot}05$) for the 3-parameter model.

A visual assessment of the comparison can be obtained from figure 2. This shows ($t$, $P$) diagrams for subjects C and J. Subject C shows the least, while subject J shows the most, improvement by adding the third parameter, apart from when the 2- and 3-parameter curves are indistinguishable for subject K.

## 5. Discussion

The first major result of note is that the introduction of a time asymptote at $t = k$ has not been vacuous. This third parameter is relevant, and has made a significant improvement in the goodness of fit to data over the standard 2-parameter CP model equation. While this parameter has a clear mathematical and graphical interpretation, its negative value may appear to pre-empt biological interpretation. This however need not be so.

Noting that $k < 0$ (figure 1), it is evident that the hyperbolic curve must cut the $t = 0$ axis at some finite power. Let us denote this point by $P = P_{\text{max}}$, in which case $P_{\text{max}}$ has the obvious biological interpretation of being the maximal 'instantaneous' power. The fitting of equation 3c below, provides the estimates of $P_{\text{max}}$ listed in table 2. These estimates, while not strictly comparable with estimates of any other studies, are significantly greater than zero ($p < 0{\cdot}001$), and with the exception perhaps of subjects K and L, are within normal maximal limits (Davies and Sandstrom 1989, Van Ingen Schenau *et al.* 1994). For similar reasons to the above, it can be seen that the standard errors of $P_{\text{max}}$ for some subjects are unduly large and the inter-subject range of $P_{\text{max}}$ is perhaps wider than desirable. The relationship between $k$ and $P_{\text{max}}$ can easily be established using equation 3b by putting $P = P_{\text{max}}$ at $t = 0$ and obtaining

$$k = \frac{\text{AWC}}{\text{CP} - P_{\text{max}}}$$

which leads to the alternate parameterization

$$t = \frac{\text{AWC}}{P - \text{CP}} + \frac{\text{AWC}}{\text{CP} - P_{\text{max}}} \tag{3c}$$

We may ask what further biological insight into the power/endurance relationship can be derived from this model. Vandewalle *et al.* (1989) has questioned the assumption of the CP test that at the point of exhaustion, all of AWC is necessarily used up. Specifically, Saltin and Karlsson (1971) found that around 90 min of cycle ergometry to exhaustion at 75% of $\dot{V}O_2$ max resulted in far more glycogen depletion than did around 29 min to exhaustion at 90% of $\dot{V}O_2$ max.

An alternate assumption may be problematic, but following the reasoning of Morton (1990), let us suppose that the maximal power $P_{\text{m}}(\tau)$ that could be produced at any instant $\tau$, is proportional to the amount of anaerobic capacity available at that instant, denoted AC($\tau$). Clearly in the fully rested state when AC($\tau$) = AWC for any $\tau$, $P_m(\tau) = P_{max}$ as outlined above. Also, as implicit in the original CP test assumptions, $P_m(\tau)$ = CP when the anaerobic capacity is fully depleted, i.e. when AC($\tau$) = 0 for any $\tau$. In the work of Saltin and Karlsson (1971) we note that $P_m(29)$ would be about 90% $\dot{V}O_2$ max and $P_m(90)$ about 75% $\dot{V}O_2$ max, and that AC (90) at 75% $\dot{V}O_{2max} <<$ AC (29) at 90% $\dot{V}O_{2max}$. Thus

$$P_m(\tau) = \mathrm{CP} + \left(\frac{P_{max} - \mathrm{CP}}{\mathrm{AWC}}\right)(\mathrm{AC}(\tau))$$

That is, when the constant power demanded is P > CP, $P_m(\tau)$ will decline with $\tau$ from $P_{max} > P$, and the point of exhaustion, reached at time $\tau = t$, occurs when $P_m(\tau) = P$.

Now since the anaerobic capacity, under these conditions, is utilized at a rate $P - \mathrm{CP}$, it follows that

$$\mathrm{AC}(\tau) = \mathrm{AWC} - (P - \mathrm{CP})\tau$$

Thus for any constant power setting $P > \mathrm{CP}$, exhaustion occurs at a time $\tau = t$ satisfying the equation

$$\mathrm{CP} + \left(\frac{P_{max} - \mathrm{CP}}{\mathrm{AWC}}\right)(\mathrm{AWC} - (\mathrm{P} - \mathrm{CP})t) = P$$

A little algebraic manipulation produces

$$t = \frac{\mathrm{AWC}}{P - \mathrm{CP}} + \frac{\mathrm{AWC}}{\mathrm{CP} - P_{max}}$$

This is precisely the non-linear 3-parameter model equation relating test data measurement of time to exhaustion $t$ in a cycle ergometer trial at constant power $P$, to the parameters CP, AWC and $P_{max}$, given in equation 3c above.

Therefore the generalization of allowing a non-zero time asymptote leads to the identification of a third parameter, $P_{max}$, which can be interpreted as maximal 'instantaneous' power; to a confirmation that all of AWC need not necessarily be consumed at exhaustion, and to support for the conjecture that maximal power depends in a very simple linear fashion on the declining anaerobic fuel reserves available.

The experimental methods to determine CP, AWC and $P_{max}$ are straightforward. Subjects would perform a series of say five constant power cycle ergometer trials, each at a different power chosen between CP and $P_{max}$. Since CP and $P_{max}$ are unknown and require to be estimated, these choices are important. It is suggested that one or two be selected at high to very high levels such that endurance times are low to very low, say down to about 1 min or a little less. This should assist in improving estimation of $P_{max}$. Likewise one or two should be at low power settings such that endurance times are long, in excess of say 15 min. This should assist in improving estimation of CP. This spreading is aimed at improving the precision of estimation (Housh *et al.* 1990) and should eliminate those few sorts of exceptions seen in the data example above. The fifth setting ought to be intermediate. These tests should be performed in a random order, and with time intervals between each to allow sufficient recovery to ensure independence between successive test durations.

The recorded times to exhaustion $t_i$ would be regressed against power demands $P_i$ by fitting equation 3c (or 3b if preferred) to the data. This can be achieved by least squares using suitable software for non-linear regression, so obtaining estimates of CP, AWC and $P_{max}$ (or $k$) and their standard errors.

## 6. Conclusions

This paper has presented a 3-parameter critical power model. It adds the parameter $P_{max}$, the maximal 'instantaneous' power, to the anaerobic work capacity and critical power parameters of the standard CP model. In so doing it appears to correct the overestimation bias in the estimation of CP and the underestimation bias in the estimation of AWC. The 3-parameter CP model does however need more selective choice of power settings and more sophisticated curve fitting procedures.

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